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A NONLINEAR MODEL TO ESTIMATE THE LONG TERM CORRELATION BETWEEN MARKET CAPITALIZATION AND GDP PER CAPITA IN EASTERN EU COUNTRIES

Abstract. *The connection between the macroeconomic development on one hand and the stock market dynamics on the other hand is the focus of many research initiatives. We are trying to apply the methodology used in the field of macroeconomic convergence to the dynamics of market capitalization for European economies. Under the general standard form of the convergence theory, which states that in the long run, as income per capita increases its corresponding growth rate will decrease, we propose a non-linear model that simulates the convergence based on data for the Central and Eastern European countries pursuing the estimation of a theoretical (hypothetical) optimal trend with respect to certain rational criteria. The model is applied on both macroeconomic variables and the market capitalization and we relate the differences that were found to the increased volatility of the latter set of data.*
Keywords: *market capitalization, GDP per capita, nonlinear models, eastern EU countries.*

JEL Classification: C51, C53, G17

1. Introduction

Academic research on the matter of connections between macroeconomic development and stock market dynamics has been mostly lead by analyses on their interconnections in the case of developed markets and generated a wide discussion on the implications and necessity for the development of these financial markets as tools for economic growth in the case of low income countries. Opinions on the instrumentality of these markets rooted many policies intending to attain sustainable economic development in the Eastern European countries in the larger framework of economic integration.

The stream of research in this area relies on one hand on studies that reflect the historical development of capital markets in the developed economies, following the idea that an analysis of the framework under which the most developed markets emerged might set a good example for the situation in the low income countries.

On the other hand a wide selection of studies embraced the perspective of analyzing the connections among the capital markets as part of an economic integration process. In the case of the European Union research indicates a significant increase of correlations among the member countries, both at geographical and industrial level. Beckers (1999) observed a statistically significant trend of increasing correlations among various industries of countries belonging to EU. In the same vein Freimann (1998) concluded that EU member states show strong correlation trend by looking at the dynamics of the most important macroeconomic variables. On the capital markets side Fratzscher (2001) investigated the integration process of European equity markets for twenty years, and showed that these markets have become highly integrated only since 1998. This high level of integration for European capital markets was mostly explained by the political movement towards EMU through the elimination of exchange rate volatility. Reviewing the dynamics of stock markets after the creation of the Eurozone, Adjaoute and Danthine (2003) concluded that European capital markets are still segmented. On the same topic, Reszat (2003) evidenced that the contribution of the common currency to financial integration has been the stronger the more national markets have in common.

Despite the increase in the correlation of returns for the stock market indices of the emerging markets with the returns of their developed counterparts, a factor which might generate reduced investments due to the lack of incentive for portfolio diversification, Levich (2001) considers that this phenomenon may really only signal a new chapter for emerging markets, probably reducing their risk and making them more certain.

During the last decades, nonlinear models proved their tractability in capturing the properties of the dynamics of economic variables. Their use is abundant throughout the scientific literature for a wide range of empirical applications. Teräsvirta (2005), Hansen (2011) and Calin et al (2014) discuss their development and main characteristics that are relevant for econometric and forecasting investigations.

In the last period, an important dose of academic attention has been given to the study of the existing linkages between macroeconomic elements and financial markets. A pioneering and seminal study in this direction was conducted by Chen, Roll and Ross (1986). The authors proved the existence of a strong dependence between macroeconomic variables such as inflation or output and the dynamics of the financial returns. This contribution led to a strong wave of academic initiatives materialized in research like: Fama (1990), Schwert (1990), Poon and Taylor (1991), or Foresti (2006).

Albu et al (2014a and 2014b) study the impact of a series of unconventional monetary policies on CDS returns. The authors observe a clear connection between the set of monetary measures known in the scientific literature as quantitative easing and the dynamics of credit risk as observed in the CDS returns.

Taking into account one of the consequences of the standard convergence theory (which states simply that in the long run as income per capita increases its growth rate decreases) and using available data, we are proposing a nonlinear model to estimate a theoretical (hypothetical) optimal trend with respect to certain rational criteria by following the stream of research mentioned above.

Specifically, we impose to the simulation model, which is operating for each constituent entity of a group, the requirement that the total estimated revenue in the last year of a period to be equal to the total actual recorded income in that year or the total estimated income of the group for the whole considered period to be equal to the total actual income of the group for the same period. In this way, the simulation model used to estimate parameters will be subject to actual statistics.

After the description of a non-linear theoretical model, we estimate its basic indicators by a recursive procedure, for a series of EU countries. Then, the study focuses on the analysis of the gap between real convergence (divergence) and optimal trend of convergence.

2. Empirical evidences in CEE compared to the theory

The standard model of the economic growth as it appears in the specific literature, states that in the long term countries are converging in terms of income per capita. One implication of such assertion is the existence of a strong negative correlation between income per inhabitant in a country and its GDP growth rate. For instance, in case of the CEE group also denoted as EU11 (comprising eleven central and eastern countries of EU) empirical data show for the period 2000-2013 a significant negative correlation between GDP per capita, y, and annual growth of GDP, μ. The graphical representation of the correlation is shown in Figure 1, where y is expressed in thousand euro PPS, μ is also computed on the base of euro PPS, i are countries in CEE (1 to 11), and t are years in the period (2000 to 2013). The estimated value for the correlation coefficient for the period was -0.457. Also, Figure 2 presents the time evolution of the two macroeconomic indicators computed as the average level in CEE (*yM_EU11* and respectively *yM_EU11*) between 2000 and 2013. This graphical representation highlights the impact of the crisis on economic growth after 2007.

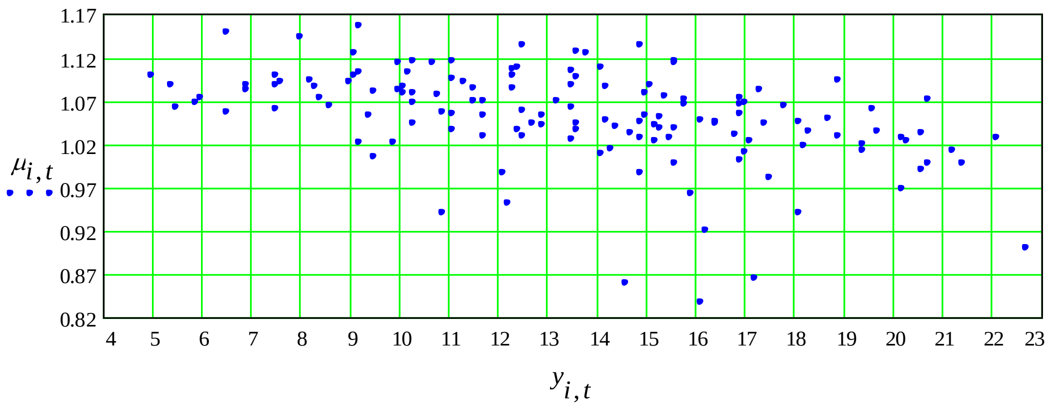


Figure 1. Correlation between GDP per capita and growth in EU11, 2000-2013

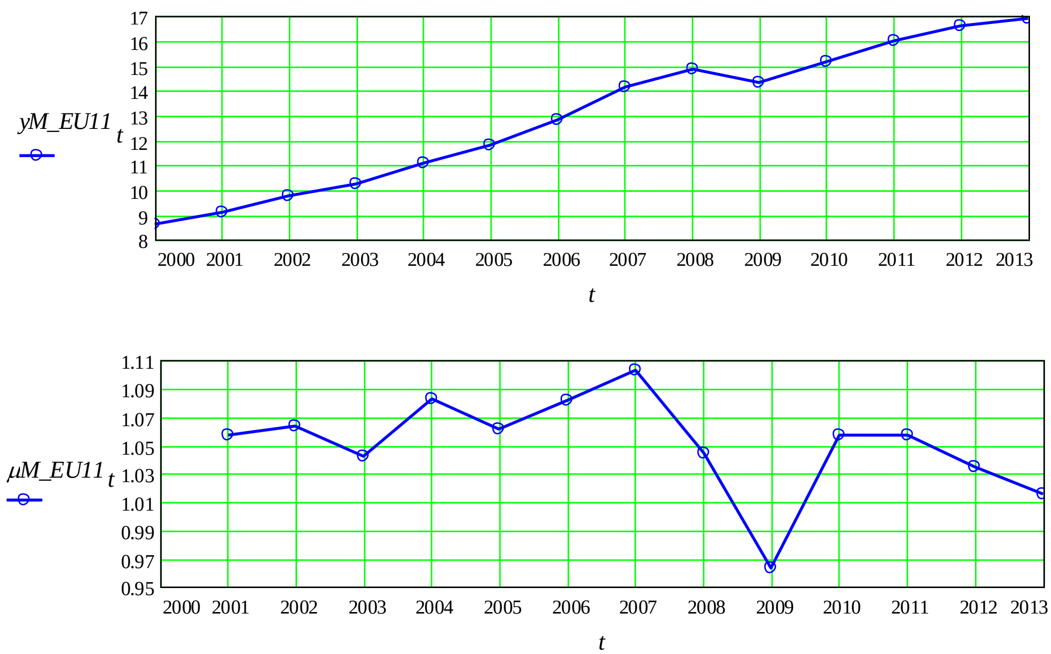


Figure 2. Dynamics of average GDP per capita and growth in EU11, 2000-2013

Moreover, there is evidently a strong negative correlation between the individual level of GDP per inhabitant and the ratio between the average GDP per capita at EU11 level and its individual level for each country, *h*, as it is reflected by the graphical representation in Figure 3. The estimated value of the correlation coefficient corresponding to the analysed period was -0.785 in this case. As we shall present later in this study, it is important to estimate the values of some essential parameters driving the long term dynamics.

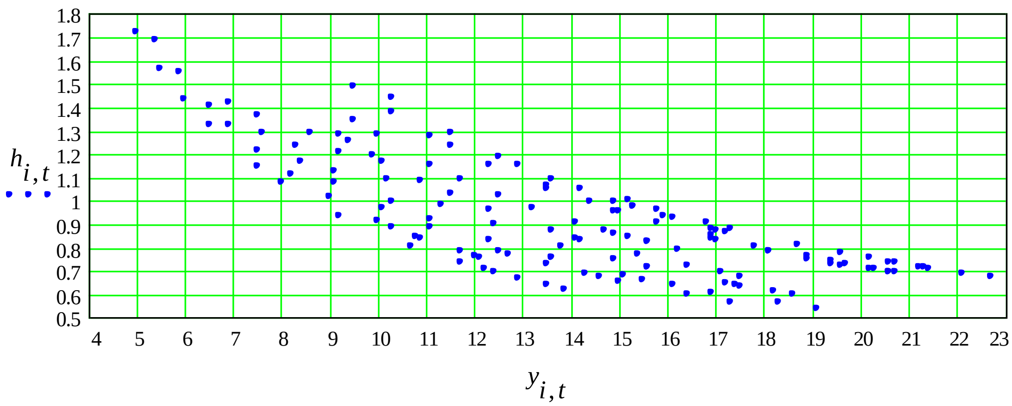


Figure 3. Correlation between GDP per capita and the ratio *h* in EU11, 2000-2013

The standard economic literature agrees on the existence of a significant positive correlation between the development level of a country and its market capitalization. Usually market capitalization is expressed as % of GDP, being published for all countries in Catalog Sources World Development Indicators. According to Standard & Poor's, "market capitalization (also known as market value) is the share price times the number of shares outstanding. Listed domestic companies are the domestically incorporated companies listed on the country's stock exchanges at the end of the year. Listed companies do not include investment companies, mutual funds, or other collective investment vehicles" (Global Stock Markets Factbook and supplemental S&P data). Contrary to this assertion, empirical data for CEE demonstrates a weak correlation between GDP per capita and market capitalization: the estimated value of the correlation coefficient using available data for the period 2000-2012 is only about +0.217. However, when we are using average data for the whole group EU11 the correlation becomes stronger (this time the estimated value of correlation coefficient is +0.404).

3. A non-linear model to simulate optimal trajectory

Taking into account the main hypotheses of the convergence theory, which derives from the standard theory of growth, and based on empirical evidences, we are proposing a non-linear model in order to simulate an optimal trajectory for GDP dynamics. The optimization criteria used assumes that the value of the total estimated (simulated) GDP at the level of the group of countries for the entire considered period is equal to real registered one.

In addition to this, a second criterion specifies that the total estimated GDP at the level of the entire group of countries for the last year of the considered period is equal to the effective level observed in that year.

Then, by using a similar simulation model we try to estimate an optimal trajectory in case of market capitalization.

A basic hypothesis of the proposed model is referring to the existence of a negative correlation between the growth index, μ, for a country in a group of countries and its level of income (or GDP) per capita, y. In the same time, a second hypothesis derives from the previous one asserting the existence of a negative correlation between the growth index, μ, for a country in a group of countries and the ratio between its income per capita and the average income per capita in that group of countries, g. Generically, the income (or GDP) index function of the two variables can be expressed in the following way:

$$\mu e(ye, ge) = [a/(ye*ge)] + 1 \qquad (1)$$

or

$$\mu e(ye, he) = (a*he/ye) + 1 \qquad (2)$$

where μe is the estimated index of income growth, a – a parameter (to be estimated), ye – the estimated level (by the model) of the average income, ge – relative proportion of individual income in average income of the group of countries, and $he=1/ge$ is the ratio between the average level of income per capita in a group of countries and the individual level of income per capita in a country. Note that, based on real data, the values of annual growth index of total income for each country, Y, is computed by its following definition relation:

$$\mu = Y_t / Y_{t-1} \qquad (3)$$

from where the real annual growth rate, r, can be obtained as

$$r = \mu - 1 = (Y_t / Y_{t-1}) - 1 \qquad (4)$$

Similarly the estimated (simulated) annual growth rate is computed as

$$re = \mu e - 1 = (Ye_t / Ye_{t-1}) - 1 \quad (5)$$

where $t-1$ and t are two consecutive years.

Based on real published data the relative gap between the individual income per person in a country, y, and the average level of it at the level of group of countries, yM, can be expressed by the following ratio

$$g = y / yM \quad (6)$$

In case of our model the estimated relative gap, ge, is computed by relation

$$ge = ye / yMe \quad (7)$$

where yMe is the estimated average value for the group of countries.

In case of real data, the value of the ratio between the average income at the level of group of countries and its individual level for a certain country can be expressed as

$$h = yM / y \quad (8)$$

but in case of our simulation model by relation

$$he = yMe / ye \quad (9)$$

In the long run, the dynamics derived from the simulation model, in line with the standard theory of growth, shows that at the limit (for very high values of GDP per capita) the values of basic macroeconomic variables reflect the following tendencies:

μ tends to 1, decreasing in case of countries for which $y > yM$ and increasing in case of those countries for which $y < yM$;

r tends to 0, decreasing when $y > yM$ and increasing when $y < yM$;

h tends to 1, increasing in case of countries for which $y > yM$ and decreasing in case of those for which $y < yM$;

g will tend also to 1, but increasing when $y < yM$ and decreasing when $y > yM$.

Based on the definition relations for derivate variables (indicators) involved in the model and on equation (1), already generally confirmed by empirical data, using a recurrence process we issued to obtain the following fundamental relation describing the GDP dynamics for each country inside of a group of countries. Thus, in case of GDP, denoted by *Y*, its dynamics, estimated by the proposed simulation model, is given by the following recurrence relation:

$$Ye_{i,t} := Ye_{i,t-1} \cdot \left[a \cdot \frac{\left(P_{i,t-1}\right)^2}{\left(Ye_{i,t-1}\right)^2} \cdot \frac{\sum_{i=1}^{n} Ye_{i,t-1}}{\sum_{i=1}^{n} P_{i,t-1}} + 1 \right] \tag{10}$$

where *Ye* is estimated GDP, *P* – total number of population, *i* – countries, *n* – total number of countries, and *t* is time (years of analyzed period for simulation). For this study, the base year was considered 2000 for which the values of variables (indicators) are identical to those computed by using real data.

Finally, based on real registered data, our model estimates an optimum value for parameter *a*. In order to apply the numerical optimization procedure we take into account two criteria:

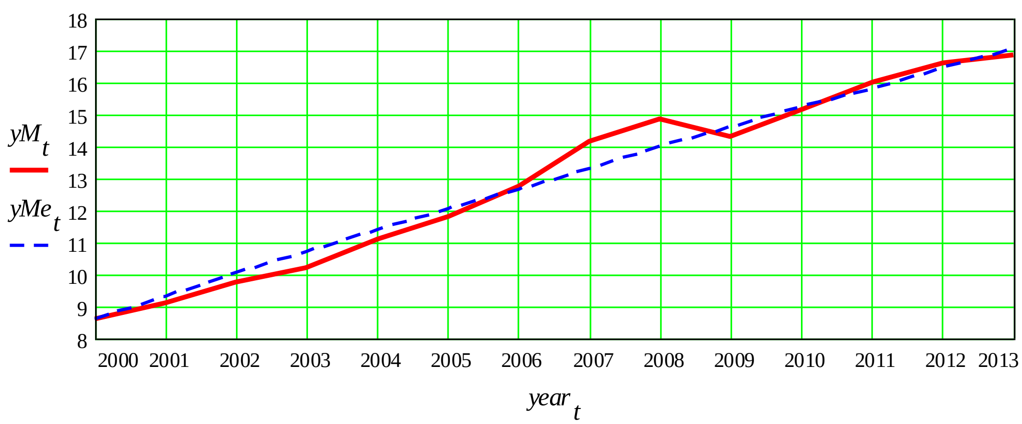
- The first imposes that the value of total estimated GDP at the level of the group of countries for the whole considered period is equal to that real registered, thus $\Sigma\Sigma Ye = \Sigma\Sigma Y$;
- The second presumes that the total estimated GDP at the level of the whole group of countries for the last year of the considered period is equal to that effectively registered in that year, thus for the last year the condition is $\Sigma Ye = \Sigma Y$.

4. Applications in case of EU11

Applying the proposed simulation model in case of CEE for GDP time series in the period 2000-2013 we obtained the following optimal values for parameter *a*: 0.6136508 in case of the first criterion and 0.5988890 in case of second criterion. Some results for the first criterion are shown as a graphical representation in Figure 4 (where *σ%y* and *σ%ye* are variation coefficients, in %, conforming to real data and respectively to simulated data, and *σ%E* is the variation coefficient between real time series and simulated time series of average GDP in EU11). The value of the correlation

coefficient between *y* and *ye* is +0.936, in case of using individual data for all countries in UE11 for all years of the period (+0.991 is the value of correlation coefficient between *yM* and *yMe*, thus in case of using time series of average level in EU11).
Without presenting detailed simulating results, we note that the proposed model can facilitate the achievement of some refined analyses of the economic dynamics that eventually permit to build a better base for economic policies in EU in order to accelerate the convergence process.
We can see a higher speed of convergence in the case of the simulating model, reflected by the gap between the two curves representing the trajectories of variation coefficients for real data and respectively simulated data (the gap between *σ%y* - *σ%ye*).

We consider useful to compare the real process of convergence within EU11 to that simulated by the model, according to the graphical representations in Figures 5. In this Figure *y1%* to *y11%* represent the real ratios (as %) between GDP per capita for each country and the average level of GDP per capita for the whole group, and *ye1%* to *ye11%* are the corresponding estimated ratios in case of simulation in order to fulfil the first optimal considered criterion (countries in EU11 ordered increasing by GDP per capita, in PPS, in the first year of the period 2000-2013 are denoted as follows: 1 – Romania, 2 – Bulgaria, 3 – Latvia, 4 – Lithuania, 5 – Estonia, 6 – Poland, 7 – Croatia, 8 – Slovakia, 9 – Hungary, 10 – Czech Rep., and 11 – Slovenia).



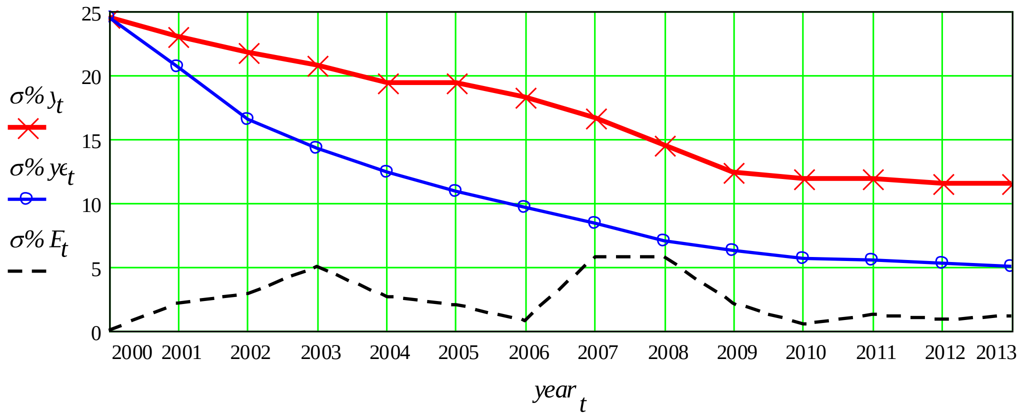


Figure 4. Simulation results in case of the first criterion for GDP in EU11, 2000-2013

In 2000 there were four countries for which the registered GDP per capita was under its average level in EU11: Romania (58.3%), Bulgaria (62.9%), Latvia (80.4%), and Lithuania (87.4%). The highest levels of GDP per capita in 2000 were registered in Czech Rep. (157.4%) and Slovenia (177.2%).

The significant changes registered during the analyzed period generally respect the rules described above: dynamics in countries with a GDP level per capita below the average level in EU11 were higher than in case of countries with a GDP per capita greater than average level. Thus, in the 2000-2013 period, the ratio between individual GDP per capita and its average level in EU11 increased by 24.0 percentage points (pp) in Romania (to 82.3%), 8.2 pp in Bulgaria (to 71.1%), 22.2 pp in Latvia (to 102.6%), 25.8 pp in Lithuania (to 113.2%), 10.1 pp in Estonia (from 100.2% to 110.3%), and 5.5 pp in Slovakia (from 110.7% to 116.2%). Contrarily, the ratio decreased by: 3.4 pp in Poland (from 107.2% to 103.8%), 18.2 pp in Croatia (from 110.7% to 92.5%), 18.1 pp in Hungary (from 120.1% to 102.0%), 35.3 pp in Czech Rep. (to 122.1%), and 50.9 pp in Slovenia (to 126.3%).

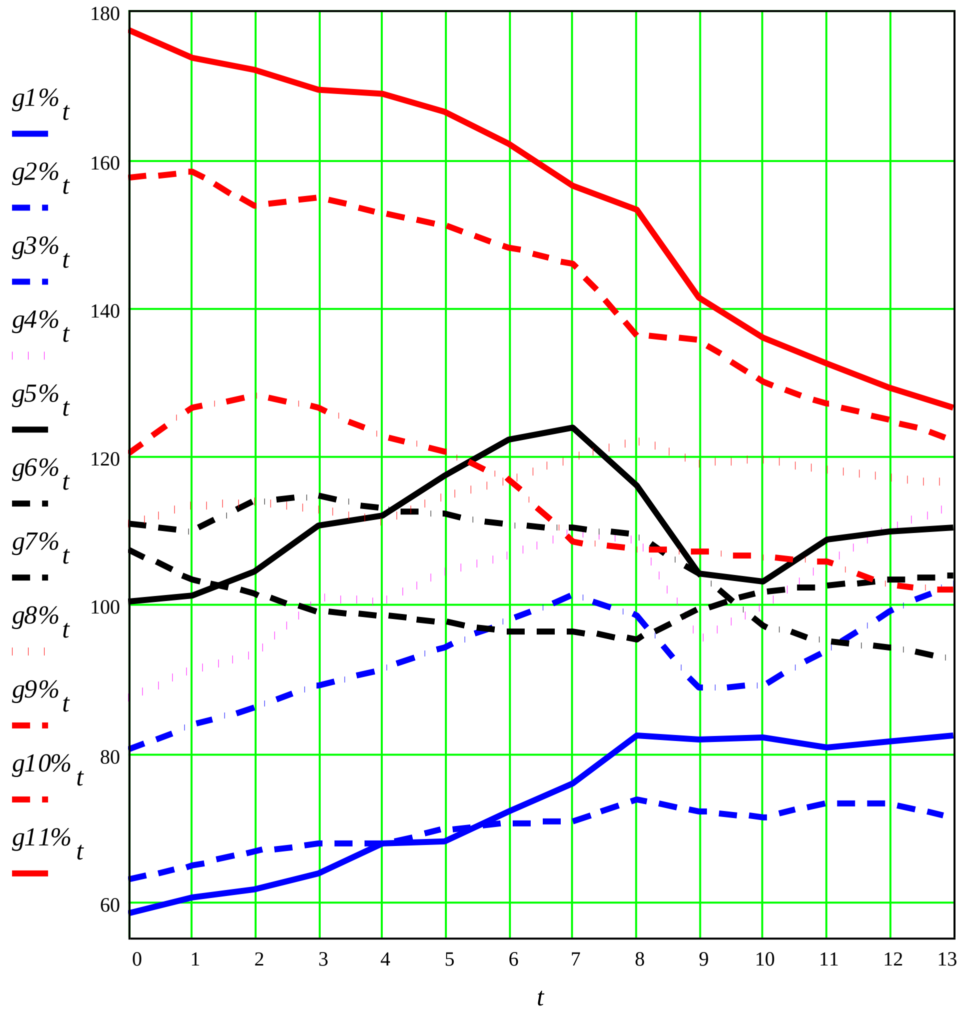


Figure 5a. Real dynamics of GDP per capita in EU11 countries, 2000-2013

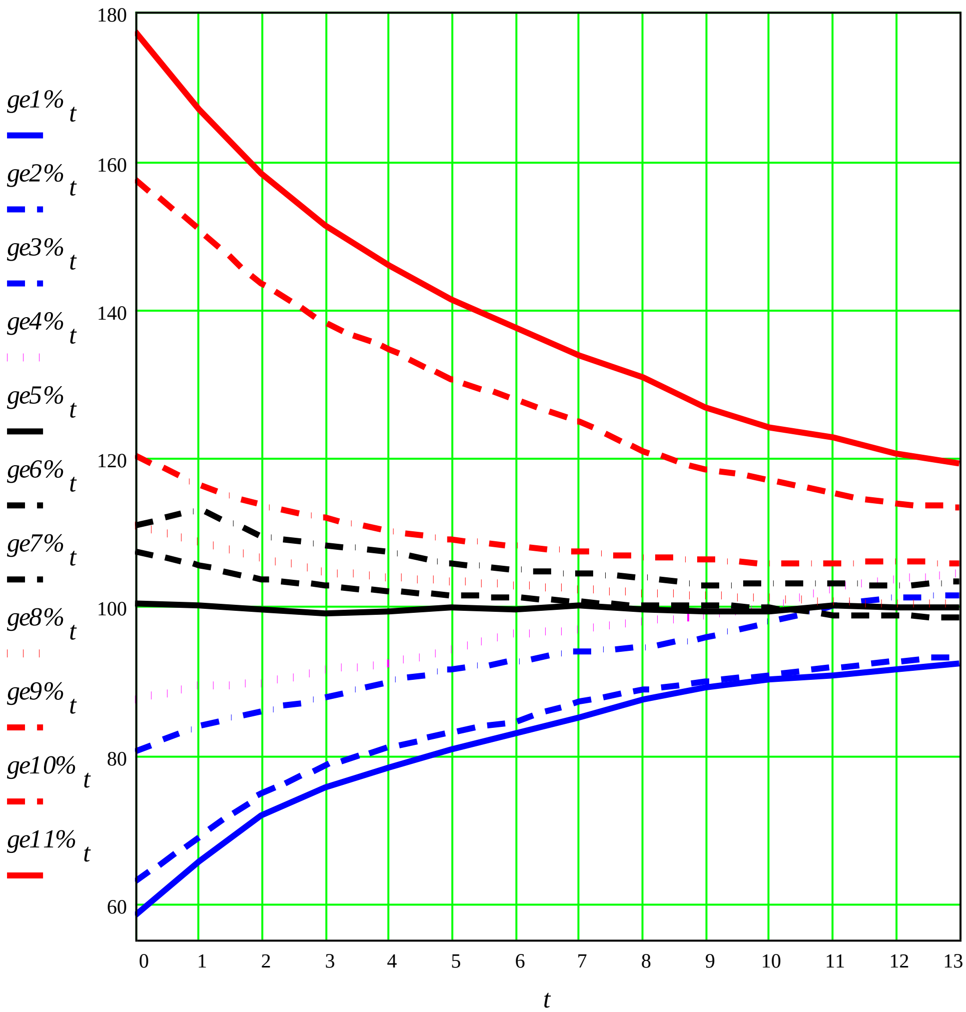
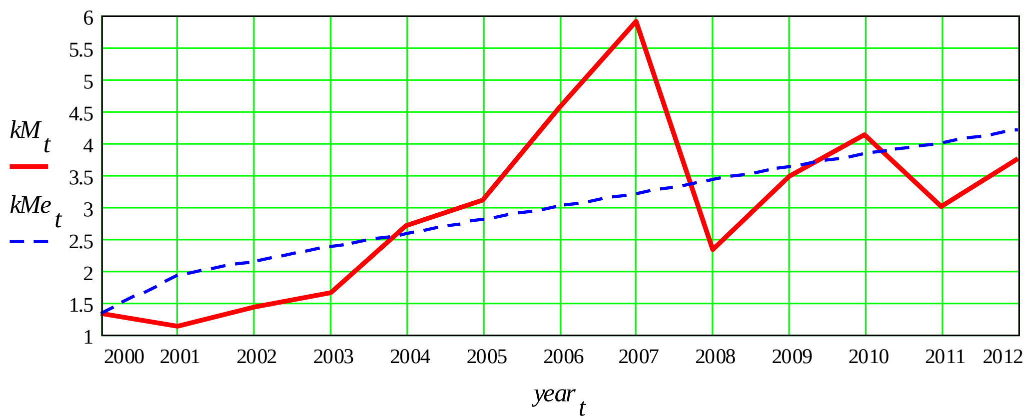


Figure 5b. Simulated dynamics of GDP per capita in EU11 countries, 2000-2013

In case of market capitalization, *K*, we used a similar simulation model to the one used for GDP, *Y*. Thus, in all relations of the model *y* (meaning GDP per capita) was replaced by *k* (meaning market capitalization per capita). By applying the simulation model in case of CEE for the available market capitalization time series in the period 2000-2013 we obtained the following optimal values for parameter *a*: 0.2017094 for the first criterion and 0.168328 for the second criterion. A series of results for the first criterion is presented as a graphical representation in Figure 6 (where *σ%k* and *σ%ke* are variation coefficients, in %, conforming to real data and respectively to simulated data, and *σ%kE* is variation coefficient between real time series and simulated time series of average market capitalization in EU11.

The value of the correlation coefficient between *k* and *ke* is +0.318, in case of using individual data for all countries in UE11 for all years of the period (+0.646 is the value of the correlation coefficient between *kM* and *kMe*, thus in the case of using time series of average level of market capitalization per capita in EU11). By comparing these results to the higher values obtained in case of GDP we observe a weaker performance of the model in the case of market capitalization. This fact shows the impact of a higher volatility in the time series specific to financial market.

The proposed model applied in case of market capitalization could support economic policies in the EU oriented towards a convergence process in mater of financial market. As in case of GDP, we can see a better convergence in case of simulating model (reflected by the gap between *σ %k* - *σ %ke*). However, contrary to the case of GDP, due to a very high volatility in case of financial market, we can see some impressive values in time series of *σ%kE*.



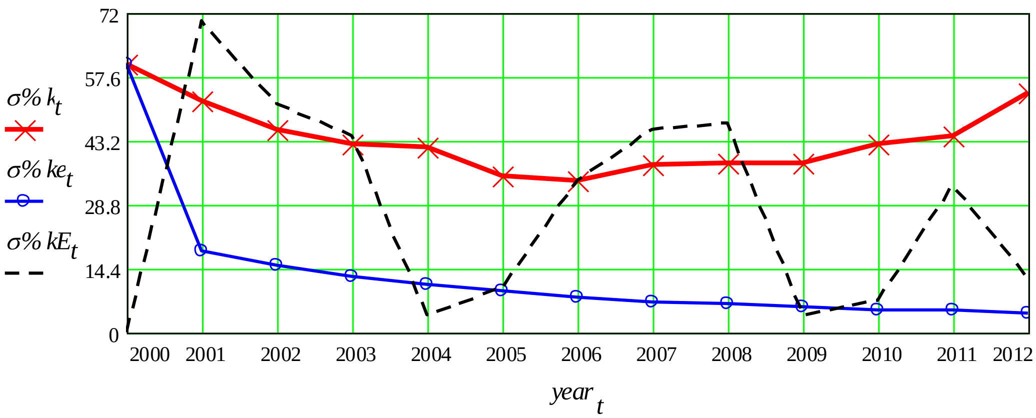


Figure 6. Simulation results in case of the first criterion for market capitalization in EU11, 2000-2012

Figures 7 show a comparison between the convergence process in mater of market capitalization within EU11 described by real data and respectively by the simulated results. In this figure, *k1%* to *k11%* represent the real ratios (as %) between market capitalization per capita for each country and the average level of market capitalization per capita in EU11, and *ke1%* to *ke11%* are the corresponding estimated ratios for the simulation according to the first optimal criterion (countries in EU11 are also ordered by considering GDP per capita in the first year of the period 2000-2013).

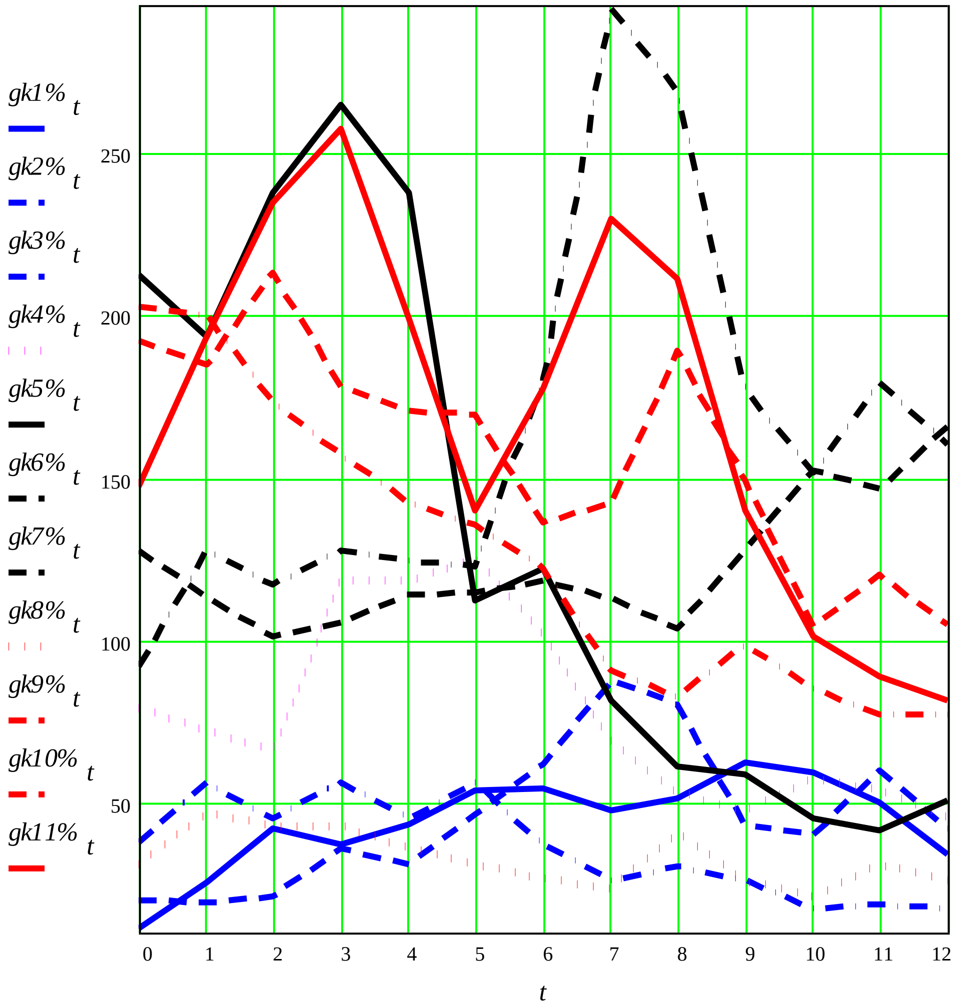


Figure 7a. Real dynamics of market capitalization per capita in EU11 countries, 2000-2012

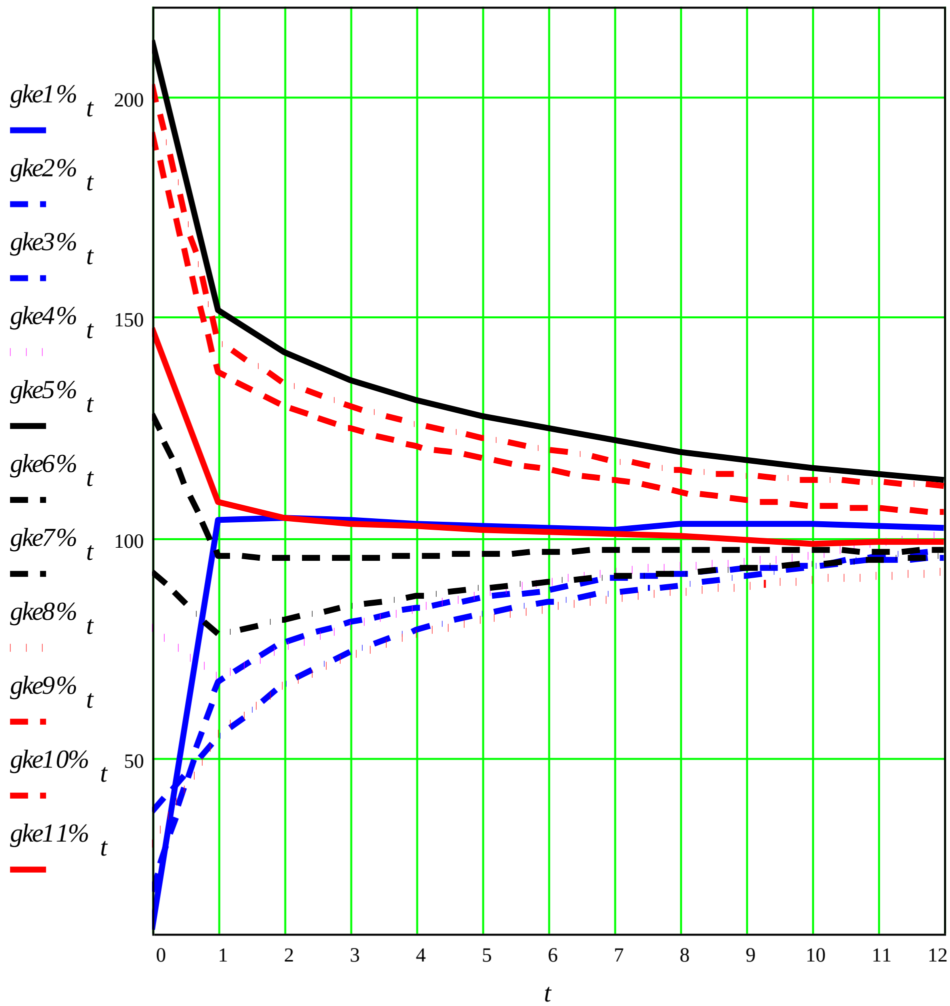


Figure 7b. Simulated dynamics of market capitalization per capita in EU11 countries, 2000-2012

5. Conclusions

The simulation model for the GDP time series in the analyzed period returns the following values for parameter *a*: 0.6136508 for the first criterion and 0.5988890 in case of the second criterion. The correlation results show a higher speed of convergence in the case of the simulation, as derived from the gap between the two curves representing the trajectories of the variation coefficients for the real data and the simulated data.

The results reveal the fact that the dynamics in countries with a GDP level per capita below the average was higher than in the case of countries with a GDP level per capita above the average level (in the EU11).

The simulation model for the market capitalization yields the following optimal values for parameter *a*: 0.2017094 in case of first criterion and 0.168328 for the second criterion. The results of the correlation coefficient show a weaker performance than in the case of GDP which could be considered a consequence of the higher volatility found in financial time series.

A future track to follow would reside in the development of an analysis on dynamics of these macroeconomic variables and the long-term volatility trends that could be extracted from the dynamics of stock market indices. According to the relevant literature in the field of macroeconomic development, the improvements of stock markets in the Eastern European region should generate growth. Therefore the perspective of increased investor confidence, instrumented by stable markets, should reflect the volatility conditions in these markets, especially due to the fact that they lack derivative instruments needed for the hedging of these types of risks.

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REFERENCES

[1] **Adjaoute, K., Danthine, J. P (2003),** ***European Financial Integration and Equity Returns: A Theory-Based Assessment***; FAME working paper;

[2]**Albu, L. L., Lupu, R., Calin, A. C., Popovici, O. C. (2014a),** ***Estimating the Impact of Quantitative Easing on Credit Risk through an ARMA-GARCH Model***; Romanian Journal of Economic Forecasting, forthcoming;

[3]**Albu, L. L., Lupu, R., Calin, A. C., Popovici, O. C. (2014b),** ***The Effect of ECB's Quantitative Easing on Credit Default Swap Instruments in Central and Eastern Europe****;* Procedia Economics and Finance, Volume 8, p. 122–128;

[4]**Beckers, S. E. (1999),** ***Investment Implications of a Single European Capital Market*****;** The Journal of Portfolio Management, 25, 3, p. 9-17;

[5]**Calin, A. C., Diaconescu, T., Popovici, O. C. (2014), *Nonlinear Models for Economic Forecasting Applications: An Evolutionary Discussion***; Computational Methods in Social Sciences, Vol II, Issue 1, p. 42-47;
[6]**Chen, N. F., Roll, R., Ross, S. A. (1986), *Economic Forces and the Stock Market***; Journal of Business, 59, p. 383-403;
[7]**Fama, E. F. (1990), *Stock Returns, Expected Returns and Real Activity***; Journal of Finance, 45, p. 1089-1108;
[8]**Foresti, P. (2006), *Testing for Granger Causality between Stock Prices and Economic Growth***; MPRA Paper No. 2962;
[9]**Fratzscher, M. (2001), *Financial Market Integration in Europe: On the Effects of EMU on Stock Markets***; Working paper No.48, European Central Bank;
[10]**Freimann, E. (1998), *Economic Integration and Country Allocation in Europe***; Financial Analysts Journal, 54, 5, p. 32-41;
[11]Hansen, B. E. (2011). *Threshold autoregression in economics*, Statistics and Its Interface, Volume 4, p. 123–127.
[12]**Levich, R. (2001), *The Importance of Emerging Capital Markets***, 4th Annual Conference on "Integrating the Emerging Market Countries into the Global Financial System", Washington, D.C., January 11-2;
[13]**Poon, S., Taylor, S. J. (1991), *Macroeconomic Factors and the UK Stock Market***; Journal of Business and Accounting, 18(5), p. 619-636;
[14]**Reszat, B. (2003), *How the European Monetary Integration Process Contributed to Regional Financial Market Integration***; HWWA Discussion Papers;
[15]**Schwert, G. W. (1990), *Stock Returns and Real Activity: A Century of Evidence***; The Journal of Finance, 45(4), p. 1237-1257;
[16]**Teräsvirta, T. (2005), *Forecasting Economic Variables with Nonlinear Models***; SSE/EFI Working Paper Series in Economics and Finance, No. 598.